Simple model of two little Higgs bosons

Witold Skiba*
Department of Physics, Yale University, New Haven, Connecticut 06520, USA

John Terning†
Theory Division T-8, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
(Received 24 July 2003; published 9 October 2003)

We construct a little Higgs boson model using a simple global symmetry group $SU(9)$ spontaneously broken to $SU(8)$. The electroweak interactions are extended to $SU(3)\times U(1)$ and embedded in $SU(9)$. At the electroweak scale, our model is a two-Higgs-boson doublet model. At the TeV scale, there are additional states, which are responsible for the cancellation of one loop quadratic divergences. We compute the effects of heavy states on the precision electroweak observables and find that the lower bounds on the masses of heavy gauge bosons and fermions are between 1 and 2 TeV.

DOI: 10.1103/PhysRevD.68.075001 PACS number(s): 14.80.Cp, 12.60.Cn, 12.60.Fr

I. INTRODUCTION

There is little doubt that the standard model of electroweak interactions is an effective theory valid below 1 TeV. New interactions will come into play around a TeV and resolve the hierarchy problem, which would be present in the low-energy theory if one tried to set the cutoff to a higher value. There are several scenarios for what the new physics at the TeV scale might be: supersymmetry, extra dimensions, dynamical symmetry breaking. Typically, weakly interacting theories at the TeV scale are in better agreement with the precision measurements of electroweak observables.

Little Higgs boson theories [1–7] provide another weakly coupled alternative for how the standard model could be embedded in a theory valid beyond 1 TeV. The little Higgs boson models employ an extended set of global and gauge symmetries in order to get rid of one-loop quadratic divergences. Without the one-loop divergences a cutoff of about 10 TeV or so is natural for a Higgs boson theory. While this may not seem like a great achievement on the road to a fundamental theory, compared for example with supersymmetry, 10 TeV is a very special scale from a practical viewpoint. The next generation of collider experiments will not be able to probe energies beyond a few TeV. It would certainly be fascinating to uncover a theory valid all the way to the Planck scale, but we will only have experimental data up to a few TeV. The energy scales above 10 TeV may remain a subject of speculation for a very long time.

Fortunately, different classes of extensions of the standard model make different predictions for the TeV spectrum. New states must enter the theory no later than a TeV to avoid fine-tuning. The cancellation of one-loop divergences in little Higgs boson theories requires new, heavy states for every divergence that is numerically significant at one loop. Thus, little Higgs boson theories predict a whole set of heavy states: vector bosons, fermions, and scalars to cancel the diagrams involving standard model gauge bosons, top quark, and the Higgs boson, respectively. Given a little Higgs boson model, it is certainly possible to make specific predictions since the theory is perturbative below the cutoff. Some more detailed studies have been reported in Refs. [8–10].

Little Higgs boson theories draw on the old idea that the Higgs boson could be a pseudo Goldstone boson [11,12]. The important recent development is the understanding of how to cleverly arrange the breaking of global symmetries. The breaking of symmetries that protect the Higgs boson mass needs to be such that the Higgs boson quartic coupling is of order one, the Higgs boson couples to fermions and gauge bosons, and at the same time one avoids quadratic divergences [1]. Symmetry breaking by interactions with numerical coefficients of order one is arranged such that individual symmetry-breaking terms do not introduce a Higgs boson mass. To get mass terms for the Higgs boson one needs more than one insertion of symmetry breaking interactions, and such diagrams are not quadratically divergent at one loop.

In what follows we present a little Higgs boson theory based on a simple global symmetry group, $SU(9)$ which is spontaneously broken to $SU(8)$. The models in Refs. [3,5] also have simple global symmetries, but the breaking patterns are different. Our model also has a simple[1] electroweak gauge group—$SU(3)\times U(1)$, analogous to Ref. [6]. As in the model with an $SU(4)\times U(1)$ gauge group [6] there is no mixing between the light and heavy charged gauge bosons induced by the Higgs boson vacuum expectation value in this theory. Our $SU(3)\times U(1)$ model has only one additional $U(1)$ compared to the standard model and thus only one Z' gauge boson. The tree-level exchanges of the single Z' result in corrections that are one half the size of those in the $SU(4)\times U(1)$ model [6,13] and thus in comparison are relatively benign for constraining the model by current data.

In the next section we describe the model. The theory is

*Email address: witold.skiba@yale.edu
†Email address: terning@lanl.gov

[1]The definition of what constitutes a simple group is different in little Higgs boson theories than it is in group theory textbooks.

based on the interactions of the Goldstone bosons associated with $SU(9)\to SU(8)$ breaking. We show how to embed $SU(3)\times U(1)$ in $SU(9)$. Our embedding produces two weak scalar doublets and two complex singlets. The model has no weak triplets. We then show how to obtain an acceptable Higgs boson quartic coupling as well as Yukawa couplings. Since the gauge symmetry is $SU(3)\times U(1)$ all left-handed fields need to come in triplets. In Sec. III we follow Refs. [13,14] and compute the tree-level effects of the heavy states on the precision electroweak data. The lower bound on the masses of the heavy states is around 2 TeV.

II. THE MODEL

Our model is based on the $SU(9)/SU(8)$ nonlinear sigma model, in which $SU(3)\times U(1)$ gauge interactions are embedded. When $SU(9)$ global symmetry is broken to its $SU(8)$ subgroup, the $SU(3)\times U(1)$ group is broken to the electroweak $SU(2)_W\times U(1)_Y$ group.

The $SU(9)/SU(8)$ coset space is described by a field Σ, which transforms linearly $\Sigma\to L\Sigma$ under $L\in SU(9)$ transformations. Σ can be expressed[2] in terms of pion fields $\Sigma=\exp(i\hat{\pi}/\sqrt{2}f)\hat{v}$, where

$$\hat{\pi}=\left(\begin{array}{cc|cc|cc} 0 & 0 & 0 & \frac{f_1}{f}h_1 & 0 & \frac{f_2}{f}h_1 \\ 0 & 0 & 0 & \frac{f_1}{f}s_1 & 0 & \frac{f_2}{f}s_1 \\ \hline 0 & 0 & 0 & \frac{f_1 f_2}{f^2}h_2 & 0 & \frac{f_2^2}{f^2}h_2 \\ \frac{f_1}{f}h_1^\dagger & \frac{f_1}{f}s_1^* & \frac{f_1 f_2}{f^2}h_2^\dagger & -2\frac{f_1 f_2}{f^2}s_2^R & -\frac{f_1^2}{f^2}h_2^\dagger & \frac{f_1^2-f_2^2}{f^2}s_2^R+is_2^I \\ \hline 0 & 0 & 0 & -\frac{f_1^2}{f^2}h_2 & 0 & -\frac{f_1 f_2}{f^2}h_2 \\ \frac{f_2}{f}h_1^\dagger & \frac{f_2}{f}s_1^* & \frac{f_2^2}{f^2}h_2^\dagger & \frac{f_1^2-f_2^2}{f^2}s_2^R-is_2^I & -\frac{f_1 f_2}{f^2}h_2^\dagger & 2\frac{f_1 f_2}{f^2}s_2^R \end{array}\right) \quad \text{and} \quad \hat{v}=\left(\begin{array}{c} 0 \\ 0 \\ \hline 0 \\ f_1 \\ \hline 0 \\ f_2 \end{array}\right). \tag{1}$$

In the equation above, s_i are electroweak singlet fields while h_i are electroweak doublets. For brevity, odd numbered rows and columns are two dimensional, while even numbered ones are one dimensional. Moreover, $f^2=f_1^2+f_2^2$ and s_2^R, s_2^I are the real and imaginary parts of singlet s_2, similarly $h_1=h_1^R+ih_1^I$ etc. The particular choice of $\hat{v}$ is meaningful because the global $SU(9)$ symmetry is explicitly broken by gauge interactions, which are described below. Otherwise we could rotate $\hat{v}$ to have a nonzero entry in only one of its components.

The pion matrix, $\hat{\pi}$, does not include all $17=80-63$ fields parametrizing the $SU(9)/SU(8)$ coset space. We chose to work in the unitary gauge, in which 5 fields that become the longitudinal components of the heavy gauge bosons are set to zero. The $SU(3)\times U(1)_X$ generators are chosen to be

$$T^a=\frac{1}{2}\left(\begin{array}{c|c|c} t^a & 0 & 0 \\ \hline 0 & t^a & 0 \\ \hline 0 & 0 & t^a \end{array}\right) \quad \text{and} \quad X=\frac{1}{3}\left(\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right), \tag{2}$$

where t^a are the $SU(3)$ Gell-Mann matrices normalized such that $\mathrm{tr}(t^a t^b)=2\delta^{ab}$. Each entry in Eq. (2) denotes a 3 by 3 block. Therefore, under the $SU(3)\times U(1)_X$ the sigma field decomposes as $\Sigma\to\mathbf{3}_{1/3}+\mathbf{3}_{1/3}+\mathbf{3}_{1/3}$. This embedding of gauge symmetry is reminiscent of the model in Ref. [15]. The generators unbroken by $\hat{v}$ are T^i for $i=1,2,3$ and $Y=(-1/\sqrt{3})T^8-X$. Under the electroweak group $SU(2)_W\times U(1)_Y$ the pion fields transform linearly, such that h_i transform as $\mathbf{2}_{-1/2}$ and s_i as $\mathbf{1}_0$. We use a nonstandard hypercharge assignment for the Higgs boson doublets h_i, opposite to the usual one. In our convention the doublets get vacuum expectation values (VEVs) in their upper components.

The transformation property of Σ, $\Sigma\to L\Sigma$, implies that the covariant derivative of Σ is

$$D_\mu\Sigma=(\partial_\mu+igT^aA_\mu^a+ig_1XX_\mu)\Sigma. \tag{3}$$

In the equation above the $SU(3)$ gauge coupling is denoted

[2]The unusual normalization of f is chosen to agree with that of the $SU(4)$ model in Refs. [6,13] so as to ease the comparison of the two models.

as g and it is equal, at tree level, to the $SU(2)_W$ coupling. The normalization of fields in Eq. (1) is such that the kinetic energy term is

$$\mathcal{L}_{kin}=(D_\mu\Sigma)^\dagger D^\mu\Sigma. \tag{4}$$

Gauge interactions induce quadratically and log-divergent terms as can be deduced by computing the Coleman-Weinberg potential. The quadratically divergent term

$$g^2\frac{\Lambda^2}{16\pi^2}\Sigma^\dagger T^a T^a\Sigma \tag{5}$$

is independent of the pion fields since T^aT^a is proportional to the identity. The same is also true for the quadratically divergent contribution arising from the $U(1)_X$ interactions. The log-divergent contribution

$$\frac{g^4}{16\pi^2}\log\left(\frac{\Lambda^2}{g^2f^2}\right)(\Sigma^\dagger T^a\Sigma)^2 \tag{6}$$

is small and generates masses for the pion fields of order $(g^2/4\pi)f$, which is of the order of the electroweak scale. Since the gauge interactions do not generate any quadratically divergent terms these interactions are not sufficient for providing the Higgs boson quartic potential.

A. Quartic potential

The underlying technique of constructing little Higgs boson theories is to arrange interactions such that each individual interaction with an $\mathcal{O}(1)$ coefficient maintains enough global symmetry to ensure that all $SU(2)$ doublets are exact Goldstone bosons. Only several interactions acting together break enough symmetry and turn Higgs boson doublets into pseudo Goldstone bosons. Interactions with small coefficients are numerically unimportant and can have an arbitrary pattern of symmetry breaking.

In order to generate a quartic potential for the two Higgs boson doublets h_i we explicitly break the $SU(9)$ global symmetry. We add two terms

$$V_q=\kappa_1(\Sigma^\dagger M_1\Sigma)^2+\kappa_2(\Sigma^\dagger M_2\Sigma)^2, \tag{7}$$

where

$$M_1=\left(\begin{array}{c|c|c}0&1&0\\\hline 1&0&0\\\hline 0&0&0\end{array}\right)\quad\text{and}\quad M_2=\left(\begin{array}{c|c|c}0&0&1\\\hline 0&0&0\\\hline 1&0&0\end{array}\right). \tag{8}$$

Each term separately preserves a different $SU(3)^3$ subgroup of global $SU(9)$. Neglecting gauge interactions, the $SU(3)^3$ symmetry is exact. In both cases $SU(3)^3$ is broken to $SU(2)^3$ by the VEV $\hat{v}$. Such breaking generates three sets of exact Goldstone bosons, which transform as doublets under $SU(2)_W$. One linear combination of these three doublets is eaten when the $SU(3)$ gauge group is broken to $SU(2)_W$. The remaining two linear combinations are two physical Higgs boson doublets, which stay exactly massless when one of the κ_i's is set to zero.

Note that this would not be the case for many other choices of symmetry breaking spurions M. For example, spurion

$$\tilde{M}=\left(\begin{array}{c|c|c}1&0&0\\\hline 0&0&0\\\hline 0&0&-1\end{array}\right) \tag{9}$$

also breaks $SU(9)$ to $SU(3)^3$. However, this particular $SU(3)^3$ subgroup is broken by $\hat{v}$ to $SU(3)\times SU(2)^2$ producing two massless doublets instead of three.

The two terms in Eq. (7) acting together break $SU(9)$ to a single $SU(3)$. This preserved $SU(3)$ is the same as the gauged $SU(3)$ whose generators are given in Eq. (2). The breaking of $SU(3)$ to $SU(2)$ yields only one doublet, which is eaten. Therefore, the uneaten Higgs boson doublets become pseudo Goldstone bosons and can obtain non derivative interactions.

Expanding V_q to quadratic order in the singlet fields and to the quartic order in doublets we obtain

$$\begin{aligned}V_q=&2\kappa_1\left(f_1\mathrm{Im}(s_1)-\frac{f_2}{\sqrt{2}f}\mathrm{Re}(h_2^\dagger h_1)\right)^2\\&+2\kappa_2\left(f_2\mathrm{Im}(s_1)+\frac{f_1}{\sqrt{2}f}\mathrm{Re}(h_2^\dagger h_1)\right)^2\\&+\mathcal{O}(s^3,h^5).\end{aligned} \tag{10}$$

Integrating out $\mathrm{Im}(s_1)$ gives the quartic interaction

$$V_{eff}=\frac{\kappa_1\kappa_2f^2}{\kappa_1f_1^2+\kappa_2f_2^2}[\mathrm{Re}(h_2^\dagger h_1)]^2, \tag{11}$$

which indeed vanishes when either κ_1 or κ_2 is zero.

The model as it is defined up to now needs to be amended, otherwise there is a potential problem. The omitted $\mathcal{O}(s^3)$ terms in Eq. (10) include the term

$$2\sqrt{2}(\kappa_2-\kappa_1)\frac{f_1f_2}{f}\mathrm{Im}(s_1)^2s_2^I. \tag{12}$$

This term generates a quadratically divergent tadpole for the field s_2^I, which means that s_2^I gets a VEV. The VEV for s_2^I rotates between the nonzero components of $\hat{v}$ in Eq. (1). If $\langle s_2^I\rangle$ is $\mathcal{O}(f)$ it will create a hierarchy between f_1 and f_2, that is either $f_1\ll f_2$ or $f_1\gg f_2$. We assumed so far in our analysis that f_1 and f_2 are of the same order of magnitude.

There are two simple ways of ensuring that $\langle s_2^I\rangle$ is small compared to f. First, we can impose a discrete symmetry that interchanges the two terms in Eq. (7). Then $\kappa_1=\kappa_2$ and the tadpole-generating term is absent. Second, we can add a potential which gives a mass to s_2^I. A linear term will force a

VEV for s_2^I, but a large enough mass term will prevent this VEV from being of order f. A suitable potential is for example

$$\begin{aligned} V_s &= \rho_1(\Sigma^\dagger N_1 \Sigma)^2 + \rho_2(\Sigma^\dagger N_2 \Sigma)^2 \\ &= 2\sqrt{2} s_2^I \frac{f_1 f_2}{f}(\rho_2 f_2^2 - \rho_1 f_1^2) + (s_2^I)^2 \\ &\quad \times \left(\rho_1 \frac{3 f_1^2 f_2^2 - f_2^4}{f^2} + \rho_2 \frac{3 f_1^2 f_2^2 - f_1^4}{f^2} \right) \\ &\quad - (\rho_2 f_2^2 - \rho_1 f_1^2) \left[\frac{f_2^2}{f^2} h_1^\dagger h_1 + \frac{f_1^2 - f_2^2}{f^2} h_2^\dagger h_2 \right] + \cdots, \end{aligned} \tag{13}$$

where

$$N_1 = \left(\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \end{array} \right) \quad \text{and} \quad N_2 = \left(\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 1 \end{array} \right). \tag{14}$$

Each term preserves an $SU(6)\times SU(3)$ subgroup of $SU(9)$, which is spontaneously broken to $SU(5)\times SU(2)$. Therefore, adding these two stabilizing terms does not introduce a large Higgs boson mass. It is clear by examining Eq. (13) that the mass terms for the Higgs boson fields vanish when the tadpole term for s_2^I vanishes. At the minimum of the potential there is no tadpole and a Higgs boson mass is not generated by the stabilizing potential.

B. Yukawa couplings

Incorporating Yukawa couplings is straightforward, and can be done in a manner outlined in Ref. [6]. Here we will show how to include the Yukawa couplings for the third family, other families of quarks and leptons can be included in the same way. Since the electroweak gauge interactions are enlarged to $SU(3)\times U(1)$ all families of quarks and leptons need to be extended to form representations of the larger electroweak group. This is not necessary in most little Higgs boson models, in which the enlarged electroweak group contains an $SU(2)\times U(1)$ subgroup. In such cases, the light families can be coupled directly to the Higgs boson doublet without regard to one-loop quadratic divergence, as this divergence is numerically unimportant for the light fermions.

We add a pair of vectorlike fermions χ_L and $\hat{\chi}_R$ such that $Q_L=(t_L, b_L, \hat{\chi}_L)^T$ transforms as an $SU(3)$ triplet. The $U(1)_X$ charges of Q_L, $\hat{\chi}_R$, $\hat{t}_R$, and b_R are $-\frac{1}{3}$, $-\frac{2}{3}$, $-\frac{2}{3}$, and $\frac{1}{3}$, respectively. It is useful to introduce projection operators into $SU(3)$ subspaces of Σ,

$$\Sigma_i = P_i \Sigma, \tag{15}$$

where $P_1=(1|0|0)$, $P_2=(0|1|0)$, and $P_3=(|0|0|1)$. Each entry in the projection operators represents a three by three matrix. In terms of the three-vectors Σ_i, the Yukawa couplings are generated by

$$\begin{aligned} \mathcal{L}_{yuk} &= \lambda_1 \overline{Q}_L \Sigma_2 \hat{\chi}_R + \lambda_2 \overline{Q}_L \Sigma_3 \hat{t}_R \\ &\quad + \lambda_b \overline{Q}_L \hat{\Sigma}_{23} b_R + \text{H.c.}, \end{aligned} \tag{16}$$

where $\hat{\Sigma}_{23}^i = \epsilon^{ijk}(\Sigma_2^*)_j(\Sigma_3^*)_k$ and $i,j,k=1,2,3$.

Expanding Eq. (16) in component fields we identify the light and the heavy right-handed mass eigenstates,

$$t_R = \frac{1}{\sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}}(\lambda_1 f_1 \hat{t}_R - \lambda_2 f_2 \hat{\chi}_R),$$

$$\chi_R = \frac{1}{\sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}}(\lambda_2 f_2 \hat{t}_R + \lambda_1 f_1 \hat{\chi}_R). \tag{17}$$

In terms of these mass eigenstates, the Yukawa interactions are

$$\begin{aligned} \mathcal{L}_{yuk} &= \sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}\, \overline{\chi}_L \chi_R - i \frac{\lambda_t}{\sqrt{2}} (\overline{t_L}, \overline{b_L}) h_2 t_R \\ &\quad - i \frac{\lambda_b}{\sqrt{2}} (\overline{t_L}, \overline{b_L}) \hat{h}_2 b_R + \cdots + \text{H.c.}, \end{aligned} \tag{18}$$

where $\hat{h}_2 = i\sigma_2 h_2^*$ and

$$\lambda_t = \frac{\lambda_1 \lambda_2 f}{\sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}}. \tag{19}$$

Note that the Yukawa coupling of the b quark could have involved $\hat{h}_1$ instead of $\hat{h}_2$. This can be accomplished by replacing $\hat{\Sigma}_{23}$ by $\hat{\Sigma}_{12}$ in the last term in Eq. (16). Therefore, the model can be either the type 1 or type 2 two-Higgs boson doublet model [16].

Cancellation of the quadratic divergences introduced by the top quark against the divergences introduced by the heavy fermion χ can be verified explicitly. However, the global symmetries of the Yukawa couplings make the absence of quadratic divergences apparent. The terms proportional to λ_1 and λ_2 in Eq. (16) separately preserve different $SU(6)\times SU(3)$ subgroups of the global $SU(9)$ symmetry. The $SU(6)\times SU(3)$ subgroups are identical to those subgroups preserved by the two terms in Eq. (13), and are broken to $SU(5)\times SU(2)$ by $\hat{v}$.

Note that the mass of the heavy partner of the top quark, $\sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}$, is not uniquely determined in terms of λ_t and f. Depending on the values of f_1/f_2 and λ_1/λ_2 one can alter the ratio of the χ mass to the top mass. This will be important in the next section, where we obtain a lower bound on f. Even with f bounded, χ can be light enough to promptly cancel quadratic divergences induced by the top quark. It is quite generic that models with several parameters will have the freedom of varying m_χ/m_{top} and this feature is also present in the model of Ref. [6].

It is easy to also incorporate lepton Yukawa couplings. We add a vector-like pair of heavy leptons in order to get a triplet of left-handed fields $L_L=(\nu_L,\tau_L,\psi_L)^T$ with X charge 1/3. The right-handed fields τ_R and ψ_R carry charges 1 and 0, respectively. The Yukawa couplings

$$\mathcal{L}_{yuk}=\lambda_2\overline{L}_L\Sigma_2\psi_R+\lambda_\tau\overline{L}_L\hat{\Sigma}_{23}\tau_R+\text{H.c.} \tag{20}$$

give mass of order f to the heavy lepton and a standard Yukawa coupling for the τ.

C. Electroweak symmetry breaking

The quartic potential displayed in Eq. (11) does not stabilize arbitrary Higgs boson VEVs. It leaves a flat direction along which $\langle h_1\rangle\langle h_2\rangle=0$. This feature is identical to the Higgs boson potential discussed in Ref. [5]. These flat directions are not problematic if the Higgs boson potential, in addition to the quartic term, includes positive Higgs boson masses and a negative B-term

$$V_{brk}=m_1^2h_1^\dagger h_1+m_2^2h_2^\dagger h_2+B(h_1^\dagger h_2+h_2^\dagger h_1), \tag{21}$$

where $m_1^2,m_2^2>0$, $B<0$. Electroweak symmetry breaking requires $B^2>m_1^2m_2^2$.

The quartic term leaves additional flat directions where $\text{Re}(h_1)=0$ and $\text{Im}(h_2)=0$, and an analogous flat direction with h_1 and h_2 interchanged, because the quartic term vanishes for $\text{Im}(h_1^\dagger h_2)=0$. These additional flat directions are not dangerous if all coefficients of the Higgs boson potential are real. This can be accomplished by imposing *CP* conservation in the Higgs boson sector. If imposing *CP* symmetry is for some reason undesirable it is easy to modify the terms in Eq. (7) to yield $V_{eff}\propto|h_2^\dagger h_1|^2$. One possibility is to modify the matrices $M_{1,2}$ defined in Eq. (8) by erasing one of the nonzero entries in each matrix. For simplicity, we will assume that all coefficients in the Higgs boson potential are real.

So far, all the interactions we have introduced preserve enough global symmetries to give two massless Higgs boson doublets. The exact symmetries are either an $SU(3)^3$ broken to $SU(2)^3$ or an $SU(6)\times SU(3)$ broken to $SU(5)\times SU(2)$. The same set of symmetries is enjoyed by operators that are induced from the tree-level Lagrangian by one-loop quadratically divergent diagrams. Therefore, none of these terms can generate a Higgs boson mass terms. It is often not apparent that the doublet mass terms are not generated by the "Higgs-friendly" operators. When expanding such operators in terms of component fields, there exist terms quadratic in Higgs boson fields accompanied by terms linear in the $SU(2)$ singlet fields. As we discussed at the end of Sec. II A, one needs to identify the proper vacuum, in which the tadpoles vanish and the Higgs boson mass terms are exactly zero.

It is therefore necessary to generate operators with less symmetry. Such operators can be included in the tree-level Lagrangian with small coefficients. Introducing new tree-level operators may not be necessary because the logarithmically divergent one-loop diagrams, as well as two-loop quadratically divergent diagrams, produce operators with less global symmetry. This is because such diagrams involve insertions of several operators each with a different symmetry structure. The diagrams involving several operators generically yield operators that preserve only a common subgroup of symmetries of all operators involved. The Higgs boson masses generated by the log-divergent diagrams and the two-loop quadratically divergent diagrams are of order $f/4\pi$, which is the desired order of magnitude.

For example, the log-divergent contribution coming from gauge interactions, Eq. (6), gives identical mass terms for h_1 and h_2. Similarly, there is a log-divergent contribution from the top quark and its heavy partner χ, which is proportional to $|\Sigma_3^\dagger\Sigma_2|^2$. This operator also generates Higgs boson masses.

III. PRECISION ELECTROWEAK MEASUREMENTS

As is well known [13,14,17,18] precision electroweak measurements can place tight constraints on new theories of electroweak symmetry breaking. The simplest method for calculating these constraints is to construct a series of effective theories by sequentially integrating out the massive modes. In the model considered here, the Σ VEV breaks $SU(3)\times U(1)$ down to $SU(2)_L\times U(1)_Y$ and gives masses of order f to some of the gauge bosons. With no Higgs boson VEVs the mass matrix is diagonalized in the basis,

$$X^\mu=\frac{-1}{\sqrt{g^2+g_1^2/3}}\left(gB_L^\mu+\frac{g_1}{\sqrt{3}}B_H^\mu\right), \tag{22}$$

$$A_8^\mu=\frac{1}{\sqrt{g^2+g_1^2/3}}\left(gB_H^\mu-\frac{g_1}{\sqrt{3}}B_L^\mu\right). \tag{23}$$

The gauge bosons A_i^μ (for $i=4,5,6,7$) and B_H^μ get masses

$$M_A=\frac{gf}{\sqrt{2}}, \tag{24}$$

$$M_{B_H}=\frac{\sqrt{2(3g^2+g_1^2)}f}{3}. \tag{25}$$

The Higgs boson VEVs introduce the usual nonlinear sigma model masses for the light $SU(2)_L\times U(1)_Y$ gauge bosons as well as (in the current model) mixing only between B_H^μ and B_L^μ. Integrating out the heavy gauge bosons and plugging in the Higgs boson VEVs

$$\langle h_1\rangle=\cos\beta\begin{pmatrix}v\\0\end{pmatrix}, \tag{26}$$

$$\langle h_2\rangle=\sin\beta\begin{pmatrix}v\\0\end{pmatrix} \tag{27}$$

we obtain the W and Z masses

$$M_W^2=\frac{g^2v^2}{4}\left(1-\frac{v^2}{6f^2}\right), \tag{28}$$

$$M_Z^2=\frac{(g^2+g'^2)v^2}{4}\left(1-\frac{(7g^4-6g^2g'^2+3g'^4)v^2}{24f^2g^4}\right). \tag{29}$$

The low-energy neutral current Lagrangian is given by

$$\mathcal{L}_{nc}=eA_\mu J_Q^\mu+\frac{e}{c_Ws_W}Z_\mu\bigg[J_3^\mu\left(1+\frac{g'^2(g^2-g'^2)v^2}{8g^4f^2}\right)+J_8^\mu\frac{\sqrt{3}(g'^2-g^2)v^2}{8g^2f^2}-s_W^2J_Q^\mu\left(1+\frac{(g'^4-g^4-g'^4)v^2}{8g^4f^2}\right)\bigg]-\frac{(3g'^2-g^2)^2}{2g^4f^2}(J_3-J_Q)^2-\frac{g'^2(3g^6-2g^4g'^2)v^2}{2\sqrt{3}g^8f^2}\times J_8(J_3-J_Q)-\frac{9g^4-6g^2g'^2+2g'^4}{12g^4f^2}J_8^2. \tag{30}$$

Note that using the relation $J_8^\mu=-\sqrt{3}(J_Y^\mu+J_X^\mu)$ this Lagrangian can be rewritten as

$$\mathcal{L}_{nc}=eA_\mu J_Q^\mu+\frac{e}{c_Ws_W}Z_\mu\bigg[J_3^\mu\left(1+\frac{(4g^2g'^2-3g^4-g'^4)v^2}{8g^4f^2}\right)+J_X^\mu\frac{3(g^2-g'^2)v^2}{8g^2f^2}-s_W^2J_Q^\mu\left(1+\frac{(4g^2g'^2-3g^4-g'^4)v^2}{8g^4f^2s_W^2}\right)\bigg]-\frac{(3g'^2-g^2)^2}{4g^4f^2}(J_3-J_Q)^2-\frac{9}{4f^2}J_X^2. \tag{31}$$

It is understood that J_X in the neutral current Lagrangian is replaced in terms of the quark and lepton currents and does not include the heavy fermion contributions. The low-energy observables in neutral current scattering are then obtained by integrating out the Z [13]. In addition, the heavy fermions $\chi_{L,R}$ introduced in Sec. II B also affect the electroweak couplings of up-type quarks and neutrinos. The mass mixing of the charge 2/3 quarks can be diagonalized by a bi-unitary transformation, and since this is a 2×2 matrix we can parametrize it by one mixing angle given by

$$\theta_{\rm mix}=\frac{v}{f}\,\eta. \tag{32}$$

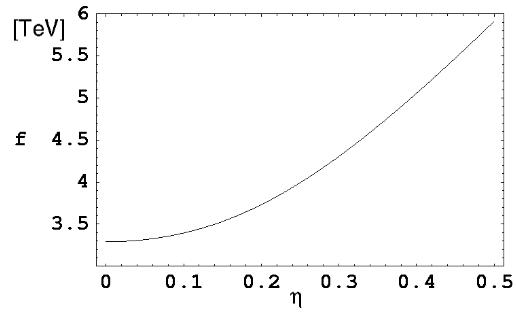

FIG. 1. 95% confidence level bound on f as a function of the mixing parameter η.

Using Eq. (16) in the massless quark limit ($\lambda_1\to0$) the parameter η is given by

$$\eta=\frac{\sin\beta}{\sqrt{2}}\frac{f_1}{f_2}. \tag{33}$$

Similarly by taking $\lambda_2\to0$ we would arrive at a result for η with f_1 and f_2 interchanged. The result for the neutrinos is identical. Thus the coupling of the up-type quarks and neutrinos to W^3 receives a correction factor $1-(v^2/f^2)\eta^2$, while the effective hypercharge of the up-type quarks becomes $(1/6)+(v^2/2f^2)\eta^2$ and the effective hypercharge of the neutrinos becomes $(-1/2)+(v^2/2f^2)\eta^2$. There are similar corrections to the couplings to $W^\pm$. The J_Q and J_X currents receive no corrections since the mixing is between two fermions with identical charges under the corresponding gauge symmetries. The neutrino mixing affects the value of G_F (in addition to the correction to the W mass),

$$G_F=\frac{1}{\sqrt{2}v^2}\left(1+\frac{v^2}{6f^2}-\frac{2\eta v^2}{f^2}\right). \tag{34}$$

To compare with experiment we use $\alpha(M_z)$, G_F, and M_Z as input parameters, and the standard definition of the weak mixing angle $\sin\theta_0$ from the Z pole [18]

$$\sin^2\theta_0\cos^2\theta_0=\frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_FM_Z^2}, \tag{35}$$

$$\sin^2\theta_0=0.23105\pm0.00008. \tag{36}$$

Expressing the precision electroweak observables in terms of these input parameters and the corrections to gauge couplings [19] implied by Eq. (31) allows us to express all the deviations of the observables from standard model predictions [20] in terms of our model parameters. As can be seen above, the dependence on $\tan\beta$ drops out of all observables, except for an implicit dependence through η. Performing a two parameter fit, we find that the weakest bound on f occurs for $\eta\sim0$, and is given by $f\geqslant3.3$ TeV at the 95% confidence level as shown in Fig. 1. It is clear that the fit to data prefers the region of small mixing. For $\eta=0.1$ the bound grows to $f\geqslant3.4$ TeV. It is interesting to note that the bound on f arises

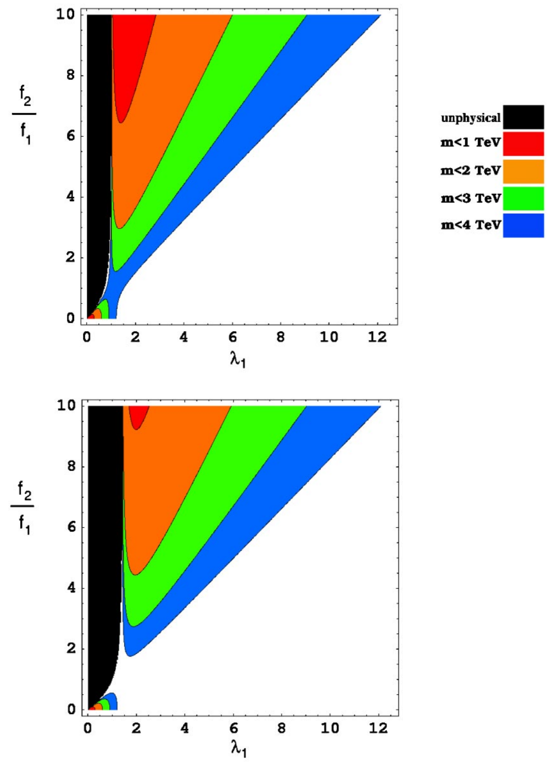

FIG. 2. Mass of the heavy fermion χ in the parameter space f_2/f_1 versus λ_1, for large $\tan\beta$ on the left and $\tan\beta=1$ on the right.

mainly from the weak charge of cesium. If this measurement is dropped from the fit the bound goes down to $f \geqslant 2.5$ TeV.

The bound $f \geqslant 3.3$ TeV corresponds to

$$M_A \geqslant 1.5 \text{ TeV}, \tag{37}$$

$$M_{B_H} \geqslant 1.8 \text{ TeV}. \tag{38}$$

There is no bound on the mass of the heavy fermion χ, since it varies from 0 to ∞ over the parameter space. The region of parameter space where χ is light is shown in Fig. 2 where we have used the constraint $\lambda_t=1$ to eliminate λ_2. Plotting the contours in this manner emphasizes large f_2/f_1 and large λ_1, plotting f_1/f_2 versus λ_2 would emphasize the opposite regime.

Since the corrections to the precision electroweak measurements were a factor of two smaller than those in Ref. [6] one might have expected that the bound on f would be a factor of $\sqrt{2}$ weaker, which, given that the bound [13] in that case was 4.2 TeV, would yield a bound of 3 TeV. The discrepancy arises from the fact that the fit for the model of Ref. [6] used four parameters, while in this case we have fit with two parameters and $\delta\chi^2$ grows with the number of fit parameters.

There may also be interesting loop corrections from the Higgs boson and heavy fermion sectors, which were recently calculated in [21] for the model of Ref. [5].

IV. CONCLUSIONS

We presented a little Higgs boson model based on a simple pattern of global symmetry breaking and at the same time avoided replicated gauge groups. In many aspects our model has similar features to the model of Ref. [6], in which the electroweak group is extended to $SU(4)\times U(1)$. The global symmetry group in our model is simple, which might yield more elegant UV completions.

A natural question concerns the possibility of the dynamical origin for the $SU(9)\to SU(8)$ breaking pattern. While the breaking pattern clearly is not a result of QCD-like dynamics, one can easily conceive of supersymmetric examples. For instance, 9 chiral superfields transforming in the vector representation of an $SO(N)$ gauge group will have an $SU(9)$ global symmetry.

We investigated how the heavy particles affect the standard model observables at the electroweak scale. At 95% confidence level, the lower bound on the pion decay constant f is 3.3 TeV. This corresponds to heavy gauge boson masses of 1.5 and 1.8 TeV, which does not require fine-tuning. The most significant fine-tuning of the Higgs boson mass results from fermion loops, and in little Higgs boson models the contribution to the Higgs boson mass from fermion loops grows with the mass of the heavy partner of the top quark. As we stressed in Sec. II B, the mass of the heavy fermion can be treated as a free parameter since it depends on the ratios λ_1/λ_2 and f_1/f_2. The ratio f_1/f_2 is constrained by the precision electroweak measurements because it contributes to the quark and neutrino mixings with $SU(2)$ singlet states. In Sec. III, we presented the bounds on the heavy fermion mass resulting from a constraint on f. The heavy fermion can be lighter than 2 TeV in a significant region of the parameter space and even as light as 1 TeV in small corners of parameter space.

The Higgs boson sector of our model is similar to those in Refs. [5,6]. The quartic term in the Higgs boson potential contains only one term, and thus it is far from being the most general two-Higgs boson doublet potential. This implies several interesting relations among the parameters of the Higgs boson sector [5]. As in all little Higgs boson models, there is a rich spectrum of TeV mass particles: gauge bosons that acquire masses from the breaking of $SU(3)$ to $SU(2)$, heavy leptons and quarks, and two complex singlet scalars. It would be exciting to see experimental confirmation of such particles, but should this happen, it will be challenging to verify that a set of heavy states comes from a little Higgs boson theory [10].

ACKNOWLEDGMENTS

We are grateful to Andy Cohen, David Kosower, and Martin Schmaltz for helpful discussions. We thank Jay Hubisz for discussions and help with the figures. W.S. is supported in part by the DOE grant DE-FG02-92ER-40704 while J.T. is supported in part by the U.S. Department of Energy under contract W-7405-ENG-36. W.S. thanks the theory group of

Boston University and the T-8 group at Los Alamos for their hospitality during enjoyable visits where part of this work was conducted. J.T. thanks the theory group at Yale for their hospitality during a stimulating visit where part of this work was done.

APPENDIX A: QUADRATIC DIVERGENCES

In this appendix we would like to outline how to explicitly check that quadratic divergences do not generate Higgs boson mass terms. So far, we used symmetry arguments so it will be reassuring to see an explicit calculation. We first present a Feynman diagram computation and then comment on the Coleman-Weinberg [22] effective potential.

The cancellations of quadratic divergences induced by the loops of fermions and gauge bosons are straightforward. More interesting is the cancellation of divergences introduced by the scalar self-interactions. In the case of scalar self-interactions there is a subtlety we would like to explain here. Some of this is certainly known to people working on this topic, but not written in the literature in any detail.

Let us concentrate on the interactions induced by the potential in Eq. (7) and for simplicity set $\kappa_1=\kappa_2=\kappa/4$ so that $V_q=\kappa/4[(\Sigma^\dagger M_1\Sigma)^2+(\Sigma^\dagger M_2\Sigma)^2]$. It is most transparent to focus on one component of the Higgs boson fields, for example $\mathrm{Re}(h_1^d)$. The superscript refers to the lower component of the Higgs boson doublet. We expand V_q in components and only display these terms that could contribute to the $\mathrm{Re}(h_1^d)$ mass term

$$V_q=\frac{\kappa f^2}{2}(s_1^I)^2+\frac{\kappa}{12}[\mathrm{Re}(h_1^d)]^2 \times\{3[\mathrm{Re}(h_2^d)]^2-4(s_1^I)^2\}+\cdots. \tag{A1}$$

The diagrams with $\mathrm{Re}(h_2^d)$ and s_1^I running in the loops do not cancel against one another and if this was the whole story they would generate a quadratically divergent mass term. The culprit is the nonlinear kinetic energy term, Eq. (4), which contains higher order terms in addition to the ordinary kinetic terms,

$$\mathcal{L}_{kin}=\frac{1}{2}(\partial_\mu\pi_i)^2-\frac{1}{12f^2}(\partial_\mu s_1^I)^2[\mathrm{Re}(h_1^d)]^2+\cdots. \tag{A2}$$

The diagram with s_1^I running in the loop with a mass insertion for this field is also quadratically divergent. It turns out that this diagram cancels the other two quadratically divergent diagrams. It might, at first, seem like a coincidence that a diagram with derivative interactions cancels potential term interactions. However, all three diagrams are proportional to the same symmetry-breaking parameter κ. Moreover, the normalization of the derivative interaction is uniquely determined by requiring that all fields have canonical kinetic energy terms.

Similarly, the higher order terms in the kinetic energy Lagrangian prevent an ingenious approach to computing the Coleman-Weinberg potential. One might be tempted to expand the Σ field around the background fields such that $\Sigma=\exp[i(\langle\hat\pi\rangle+\hat\pi)/f]\hat v$. The background fields $\langle\hat\pi\rangle$ inserted into the higher order terms in the kinetic energy spoil canonical normalization. A convenient way around this is to parametrize the nonlinear field as

$$\Sigma=\exp[i\langle\hat\pi\rangle/f]\exp[i\hat\pi/f]\hat v \tag{A3}$$

so the background fields drop out of the kinetic energy term.

Using the form (A3) we calculate $\mathrm{tr}\,\mathcal{M}^2(\langle\hat\pi\rangle)$ keeping $\langle\hat\pi\rangle$ to all orders. We get $\mathrm{tr}\,\mathcal{M}^2\propto\kappa[\sin(\langle\mathrm{Re}(h_1^d)\rangle)^2+\cos(\langle\mathrm{Re}(h_1^d)\rangle)^2]$, which is independent of $\mathrm{Re}(h_1^d)$. This would not be the case if we used a parametrization in which the kinetic energy depends on the background fields.

[1] N. Arkani-Hamed, A. Cohen, and H. Georgi, Phys. Lett. B **513**, 232 (2001).
[2] N. Arkani-Hamed, A. Cohen, E. Katz, A. Nelson, T. Gregoire, and J.G. Wacker, J. High Energy Phys. **08**, 021 (2002).
[3] N. Arkani-Hamed, A. Cohen, E. Katz, and A. Nelson, J. High Energy Phys. **07**, 034 (2002).
[4] T. Gregoire and J. Wacker, J. High Energy Phys. **08**, 019 (2002).
[5] I. Low, W. Skiba, and D. Smith, Phys. Rev. D **66**, 072001 (2002).
[6] D.E. Kaplan and M. Schmaltz, hep-ph/0302049.
[7] S. Chang and J.G. Wacker, hep-ph/0303001.
[8] N. Arkani-Hamed, A. Cohen, T. Gregoire, and J. Wacker, J. High Energy Phys. **08**, 020 (2002).
[9] R.S. Chivukula, N. Evans, and E.H. Simmons, Phys. Rev. D **66**, 035008 (2002).
[10] G. Burdman, M. Perelstein, and A. Pierce, Phys. Rev. Lett. **90**, 241802 (2003); T. Han, H.E. Logan, B. McElrath, and L.T. Wang, Phys. Rev. D **67**, 095004 (2003); Phys. Lett. B **563**, 191 (2003).
[11] D.B. Kaplan and H. Georgi, Phys. Lett. **136B**, 183 (1984); D.B. Kaplan, H. Georgi, and S. Dimopoulos, *ibid.* **136B**, 187 (1984); H. Georgi, D.B. Kaplan, and P. Galison, *ibid.* **143B**, 152 (1984).
[12] H. Georgi and D.B. Kaplan, Phys. Lett. **145B**, 216 (1984); M. Dugan, H. Georgi, and D.B. Kaplan, Nucl. Phys. **B254**, 299 (1985).
[13] C. Csaki, J. Hubisz, G.D. Kribs, P. Meade, and J. Terning, Phys. Rev. D **68**, 035009 (2003).
[14] C. Csaki, J. Hubisz, G.D. Kribs, P. Meade, and J. Terning, Phys. Rev. D **67**, 115002 (2003); J.L. Hewett, F.J. Petriello, and T.G. Rizzo, hep-ph/0211218.
[15] D.A. Kosower, Nucl. Phys. **B294**, 244 (1987).
[16] J.F. Gunion, H.E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, MA, 1990).
[17] M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991);

B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990).
[18] M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992).
[19] C.P. Burgess, S. Godfrey, H. Konig, D. London, and I. Maksymyk, Phys. Rev. D **49**, 6115 (1994).
[20] J. Erler (private communication); hep-ph/0005084.
[21] T. Gregoire, D. Smith, and J. Wacker, hep-ph/0305275.
[22] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).